A NOTE ON AN ITERATIVE APPROACH TO MULTICOPY TRAFFIC ASSIGNMENT*

F.A. Tillman[1]

Kansas State University

Manhattan, Kansas

INTRODUCTION

In an earlier paper (1) a method was proposed for solving the multicopy traffic assignment problem for urban street and highway networks. The methods becomes computationally difficult for more than two copies. Therefore the following approach is proposed for solving this problem. The problem is restated to expedite the discussion.

To realistically represent travel times a linear travel time function is proposed as follows:

$$t = a_1 + a_2 V \tag{1}$$

where

t = link travel time in hours per vehicle

a_1 = constant representing travel time at free flow conditions

a_2 = empirically derived constant

V = link volume in vehicles per link per hour.

The total travel time spent on each link is now obtained by multiplying equation (1) by the traffic volume V. Thus, the equation for total link travel time is as follows:

$$t' = a_1 V + a_2 V^2 . \tag{2}$$

---

*This research work was supported by GK818 and NASA NsG692

[1]Associate Professor and Head, Industrial Engineering Department

Thus, a linear link travel time function produces a non-linear total travel time function. A copy is defined in terms of destinations, thus a problem with three principle destinations is termed a three copy network regardless of the number of origins.

Problem Formulation

To formulate the problem, the following notations are used and which represent quantities for each copy:

(n,m) = represents the nodes $(n = 0,1,2,\ldots,N;\ m = 0,1,2,\ldots,M)$

$V^{(n,m)}$ = the number of vehicles entering at the node (n,m)

$X_H^{(n,m)}$ = the number of vehicles traveling in the horizontal direction from the node (n,m) towards node $(n,m+1)$

$X_V^{(n,m)}$ = the number of vehicles traveling in the vertical direction from the node (n,m) towards node $(n+1,m)$

$P^{(n,m)}$ = fraction of the vehicles entering node (n,m) that leave on the horizontal link

$Z^{(n,m)} = X_H^{(n,m-1)} + X_V^{(n-1,m)} + V^{(n,m)}$, the total number of vehicles at the node (n,m).

To include the effects of other copies the following terms are defined:

$VH^{(n,m)}$ = the total number of vehicles on the same links in the same direction as $X_H^{(n,m)}$, on all other copies

$VV^{(n,m)}$ = the total number of vehicles on the same links in the same direction as $X_V^{(n,m)}$, on all other copies.

Using the above notations the fraction of the volume of traffic at the node (n,m) which travels in the hroizontal direction can be expressed as:

$$P^{(n,m)} = X_H^{(n,m)} \,/\, Z^{(n,m)} \tag{3}$$

and consequently,

$$1 - P^{(n,m)} = X_V^{(n,m)} \,/\, Z^{(n,m)} \tag{4}$$

The total time required to travel the network is given by:

$$T = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{H1}^{(n,m)}[X_H^{(n,m)} + VH^{(n,m)}] + a_{H2}^{(n,m)}[X_H^{(n,m)} + VH^{(n,m)}]^2$$

$$+ a_{V1}^{(n,m)}[X_V^{(n,m)} + VV^{(n,m)}] + a_{V2}^{(n,m)}[X_V^{(n,m)} + VV^{(n,m)}]^2 \qquad (5)$$

where

$P^{(n,M)} = 0.0$ and therefore $X_H^{(n,M)} = 0.0$, $(n = 0,1,2,\dots,N)$

$P^{(N,m)} = 1.0$ and therefore $X_V^{(N,m)} = 0.0$, $(m = 0.1,2,\dots,M)$,

and where $a_{V1}^{(n,m)}$, $a_{V2}^{(n,m)}$ = the constants associated with the vertical streets from the node (n,m) to $(n+1,m)$

$a_{H1}^{(n,m)}$, $a_{H2}^{(n,m)}$ = the constants associated with the horizontal streets from the node (n,m) to $(n,m+1)$.

To summarize, the problem is one of minimizing T, given by equation (5), by finding suitable values of $P^{(n,m)}$ $(n = 0,1,2,\dots,N;\ m = 0,1,2,\dots,M)$ for each copy.

Solution Procedure

The determination of an optimal assignment for copy one of a street network is accomplished as follows:

STEP 1: Label the destination as node (N,M) and divide the network into K stages in the following manner:

K^{th} stage: All the routes that form a rectangle whose diagonal is formed by nodes $(N-1,M-1)$ and (N,M).

$(K-1)^{th}$ stage: All the routes that form a rectangle whose diagonal is formed by nodes $(N-1,M-2)$ and (N,M).

$(K-2)^{th}$ stage: All the routes that form a rectangle whose diagonal is formed by nodes (N-2,M-1) and (N,M) and any other routes that were included in the previous stages.

. . . .

. . . .

1^{st} stage: All the routes that form a rectangle whose diagonal is formed by nodes (0,0) and (N,M) and any other routes that were included in the previous stages.

Figure 1 shows a step by step procedure of dividing a 3 x 3 network into nine stages. It is noted that the nodes covered with hatch marks should be included in the stages but are excluded in the computation since they do not alter the value of $P^{(n,m)}$ when determining the minimum travel time. If the destination is an interior node, the problem is divided into quadrants using this node as the center. Each quadrant is considered a separate problem where the link volumes on the boundary links from the other quadrants are considered as inputs to the quadrant being studied. These inputs are $VH^{(n,m)}$ and $VV^{(n,m)}$ in equation (5).

STEP 2: Assume an initial value, for example 0.5, for all $P^{(n,m)}$, the fraction of vehicles at node (n,m) that travel in the horizontal direction towards the node (n,m+1) with the following exceptions:

$$P^{(n,M)} = 0.0, \; n = 0,1,2,\ldots,N$$

and

$$P^{(N,m)} = 1.0, \; m = 0,1,2,\ldots,M \; .$$

For the first copy set $VH^{(n,m)} = VV^{(n,m)} = 0$. The succeeding values are determined from $X_H^{(n,m)}$ and $X_V^{(n,m)}$ from the other copies.

STEP 3: With these values of $P^{(n,m)}$, start from the node (0,0) and determine the number of vehicles on all the routes. This is done for all nodes (n,m) by the following recursive relationships

$$\begin{aligned} X_H^{(n,m)} &= P^{(n,m)} \cdot Z^{(n,m)} \\ X_V^{(n,m)} &= (1 - P^{(n,m)}) \cdot Z^{(n,m)} \\ &= Z^{(n,m)} - X_H^{(n,m)} . \end{aligned} \tag{6}$$

These numbers must be integer, if not, they are rounded to the nearest integer.

STEP 4: With the vehicles loads as determined in step 3 by equation (6), start at the k^{th} stage, which represents node (i,j) and by keeping the number of vehicles entering the k^{th} stage $Z^{(i,j)}$ constant, determine the new value of $P^{(i,j)}$ at this node that minimizes the total travel time for this stage by changing $X_H^{(i,j)}$ (or $X_V^{(i,j)}$) in equation (7) which is obtained from equation (5),

$$\begin{aligned} T = \sum_{m=j}^{M} \sum_{n=i}^{N} a_{H1}^{(n,m)}[X_H^{(n,m)} + VH^{(n,m)}] + a_{H2}^{(n,m)}[X_H^{(n,m)} + VH^{(n,m)}]^2 \\ + a_{V1}^{(n,m)}[X_V^{(n,m)} + VV^{(n,m)}] + a_{V2}^{(n,m)}[X_V^{(n,m)} + VV^{(n,m)}]^2 \end{aligned} \tag{7}$$

The previous value of $P^{(i,j)}$ is replaced with the new value of $P^{(i,j)}$ and the number of vehicles on all the routes at this stage are adjusted according to equation (6) for:

$(n = i, i+1, \ldots, N;\ m = j, j+1, \ldots, M)$.

Note that only one value of $P^{(n,m)}$, that is $P^{(i,j)}$, is adjusted at each stage. Proceed to the next stage and repeat the process. The process is repeated for all stages until new values for all $P^{(n,m)}$ have been determined.

STEP 5: One iteration is now completed. The values of $P^{(n,m)}$ from this iteration are compared to the corresponding values from the previous iteration. When the values of $P^{(n,m)}$ do not differ significantly on two successive iterations the answer is considered optimal for this copy. If they do differ significantly, go to step 3 using these new values of the $P^{(n,m)}$ as the initial values and repeat the procedure until an optimal solution is obtained for this copy.

STEP 6: The above procedure, step 1 thru step 5, is repeated for the remaining copies where the new values of $VH^{(n,m)}$ and $VV^{(n,m)}$ are computed from $X_H^{(n,m)}$ and $X_V^{(n,m)}$ from the previous copies. One pass is completed when a solution for each copy has been obtained. The entire procedure is repeated as many times as necessary starting with the first copy until the total travel time for all copies does not differ significantly on two successive passes. When this occurs an optimal solution has been obtained.

The method proposed in (1) for solving the multicopy traffic assignment problem becomes computationally difficult when there are several nodes and copies. The proposed method provides a practical method for obtaining the optimal solution to these problems.

References

(1) Tillman, F.A., D. Pai, M.L. Funk and R.R. Snell, "An Iterative Approach to Traffic Assignment." Transportation Research, Vol 2, No. 1.

(2) Funk, M.L., R.R. Snell and J.B. Blackburn", Optimal Allocation of Trips to a Street Network". Journal of the Highway Division, ASCE Vol. 93, No. HW2, November 1967, 95-113.

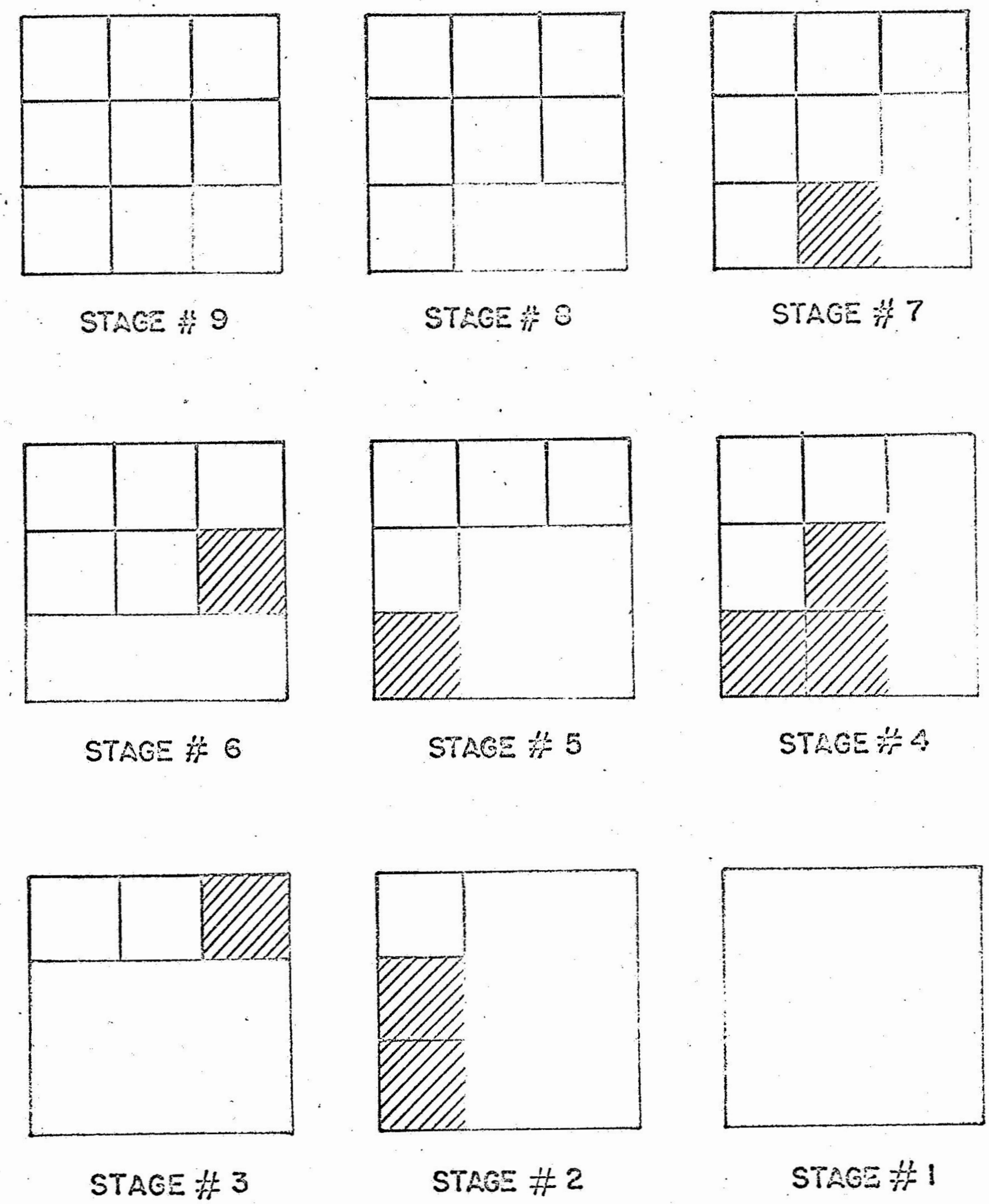

Fig. 1.. 9 stages of a 3x3 network.